

Commentary on the Statistical Estimation of Gambling Outcomes

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ABSTRACT

A review of the literature affirms the popularity of casinos and online betting. Despite the dangers inherent in gambling, players are attracted to games of chance for excitement and because of the potential for monetary rewards. Of course, the potential for monetary losses is greater than the potential for gains, and the wise player will study the risk/reward ratio to minimize losses. In this industry commentary article, we use applications of house edge in casinos, and overround in sports betting, to demonstrate ways that gambling institutions ensure a high gross profit margin.

Keywords

Statistical estimation, odds, house advantage, overround

INTRODUCTION

Casinos and other gambling establishments have become some of the most profitable businesses in the world, an accomplishment based on mathematics and gaming strategies. To gain an advantage over players, the creators of the gambling market devised the algebraic approaches known as house edge in casinos and overround in sports betting. These approaches are incorporated in game designs so that players lose money in the long run. Quite often the house edge and overround advantages are small, but over time they can result in large profits for the businesses. An examination of how gambling businesses ensure a profit can shed light on the players' motivations, behaviors, and risks.

INDUSTRY CONTEXT

Recent research reveals that, despite the clear dangers of gambling, approximately 26% of the world's population gambles (Wise, 2023). In the United States, 85% of poll respondents reported gambling at least once in their lifetime. While 30% of respondents said that gambling is fair and 62% believed they should be able to gamble whenever they want, others recognized the dangers and expressed the view that there are too many opportunities to gamble (79%) or that gambling machines should have stake limits on them (41%). Those who admitted to continuously playing no matter how well or poorly they were doing made up 0.7% of the participants. Another 1.1% considered themselves to be at medium risk of losing control, and 2.4% felt they had a low risk of losing control. Though only 22% of colleges have gambling policies, gambling is prevalent in

colleges, with approximately 67% of students involved in sports betting and 75% involved with gambling in general. The challenge faced by the gambling industry is to promote their entertainment value while balancing their potential for profit with their potential to harm individuals and the communities they serve. One way to understand how the industry functions and relates to players is to analyze the statistical estimation of gambling outcomes. As we discuss below, casinos and other gambling institutions use sophisticated approaches to manipulate the odds in their favor. Knowledgeable players can assess the odds, as well as their own risk tolerance, and make wise decisions that safeguard their personal finances.

DISCUSSION

In popular casino games, a player's expected winnings percentage (EW) is calculated by the following equation, where “factor” is the amount of money the gambler may win or lose (Nath, 2020).

$$EW = ((\text{probability of winning})(\text{winning factor}) - (\text{probability of losing})(\text{losing factor})) \times 100$$

Negating the equation above shows the percentage the casino can expect to win, which is referred to as the house advantage or house edge (HE). The house edge is calculated as follows:

$$HE = ((\text{probability of losing})(\text{losing factor}) - (\text{probability of winning})(\text{winning factor})) \times 100$$

To demonstrate the calculation of house edge, consider a player who bets one dollar that the roll of a die will result in a three. With a fair die, the player has a $\frac{1}{6}$ chance of winning and a $\frac{5}{6}$ chance of losing. In a game that does not favor the player or the house, the player wins five dollars when a three is rolled and the player loses one dollar when any of the other five possible outcomes occurs. This results in a house edge of 0%.

$$HE = ((\frac{5}{6})(1) - (\frac{1}{6})(5)) \times 100 = (\frac{5}{6} - \frac{5}{6}) \times 100 = 0\%$$

With no house edge, a casino can expect to neither win nor lose money. However, a casino's goal is to have a house advantage, so they set the winnings to favor the casino. In the above scenario, for example, the player might only be awarded four dollars, rather than five dollars, when a three is rolled. This would give the casino a house edge of 16.67%.

$$HE = ((\frac{5}{6})(1) - (\frac{1}{6})(4)) \times 100 = (\frac{5}{6} - \frac{2}{3}) \times 100 = 16.67\%$$

A house edge of 16.67% means the casino can expect to win \$16.67 if the player bets one dollar on each play and plays 100 times.

To ensure their winnings, casinos incorporate a positive house edge in almost every game they offer. One of the most popular games for players is the slot machine, which generates 65%-80% of a casino's income (Schwartz, 2018). Though most players lose money, large amounts of money are won by the lucky few who play the right machine at just the right time. Large winnings are possible, in part, because the law requires that slot machines pay out a set percentage of the bets made on the machine. The required percentage in Nevada, for example, is 85%. The system benefits the casinos because when one player wins big, others notice and are motivated to keep playing.

Unfortunately, the potential for a big win can also lead to big losses and may result in an individual developing a gambling addiction. Addicts' losses may motivate them to play more, to get the winning sensation again and to avoid leaving the casino with less money in their pocket than when they walked in (Neiger, 2023). This is a danger of the house edge, as it ensures players will statistically lose money in the long run, which can create an even greater desire to keep playing. Players who find themselves unable to leave the casino are likely to lose more money, and the cycle repeats. Problem gamblers only make up a small percentage of those one would see on a trip to a casino, but their losses are estimated to make up about a quarter of the casino's profit (Minnesota Department of Human Services, 2018).

The house edge percentage varies by game. The 16.67% house edge calculated in the example above is a high percentage, but there are several games that have even higher percentages. The largest house edges are found in a subcategory of Sic Bo at 33.33%, in Keno with up to 29%, and in Big Six with up to 24.07% (Shackleford, 2019). Higher percentages can come from different subcategories of each game as well. Big Six, for example, is a specific type of card game, but the house edge changes according to how much the gambler bets. The house edge is 11.11% if the player bets \$1, and it increases to 24.07% if the player bets \$20. The game craps has at least 18 different subcategories, with Any Seven being the subcategory with the highest house edge at 16.67%.

Though casinos win more money from games with high house edges, they offer many games for which the house edge is less than 3%. The games with low percentages are some of the most popular games, so what they lack in house edge they make up for in repetitive usage. Blackjack, for example, is an extremely popular game, therefore it is profitable for casinos despite its meager 0.28% house edge (Shackleford, 2019).

American roulette is another popular casino game. With this game, the gambler's goal is to guess the slot in a roulette wheel where a marble will drop. The wheel is spun in one direction while the marble is rolled in the other direction around the wheel's perimeter. The wheel slopes down toward its center, and as the wheel slows, the marble drops and lands in one of 38 numbered slots. In the ranges 1-10 and 19-28, the odd numbered slots are red and the even ones are black. In the ranges 11-18 and 29-36, the odd numbered slots are black and the even ones are red. Two additional slots labeled 0 and 00 are green. ("American Roulette," 2023). Consider a player who bets that the marble will land in a red slot, thus having an 18 out of 38 (about 47%) chance of being correct. Another player might bet that the marble will land in a green slot, thus having only a 2 out of 38 (about 5%) chance of being correct. The smaller the chance of a player being correct, the greater the winnings if the marble lands in a corresponding slot.

Suppose a person bets one dollar on an even number. As there are 18 even numbers and 38 slots total, there is about a 47% chance of guessing correctly and a 53% chance of guessing incorrectly. Since the house edge is calculated by: $HE = ((\text{probability of losing})(\text{losing factor}) - (\text{probability of winning})(\text{winning factor})) \times 100$, the calculation becomes: $((20/38)(1) - (18/38)(1)) \times 100 = (20/38 - 18/38) \times 100 = 5.26\%$. If a \$1 bet is placed each time and the game is played 100 times, a player can expect to lose about \$5.26 to the casino.

The design of the game is critical to the calculation of house edge. In American roulette, if the only colors were red and black, the only numbers were odd and even, and a dollar bet could win a dollar, there would be no house advantage. The 0 and 00 spaces, which are both green, are what adjust the players' chances of winning from a 50-50 game to a game that favors the casino. As a comparison, consider a coin toss, which is a 50-50 game. There is a 50% chance that a toss will

result in heads and a 50% chance that a toss will result in tails. In the long run, then, heads and tails will occur about the same number of times.

European roulette is very similar to its American counterpart, but the European roulette wheel has only 37 slots rather than 38. While American roulette has two green slots (0 and 00), European Roulette has only one (0) ("American Roulette," 2023). If a player bets on even numbers in European roulette, the calculation becomes: $((19/37)(1) - (18/37)(1)) \times 100 = (19/37 - 18/37) \times 100 = 2.7\%$.

This shows that the absence of one space (green 00) causes the house edge to decrease. In this case, if a player bets one dollar on even and plays 100 times, the expected loss is \$2.70. The expected loss for the same bet in American roulette is \$5.26.

The difference in outcome is due to the fact that there are fewer green spaces in the European version, which increases the player's probability of winning. Suppose neither 0 nor 00 were spaces and a gambler bet on even. This would mean there would be a 50% chance of a win since there are the same number of even spaces as odd spaces. However, when both 0 and 00 are included as options, a player who bets on even will lose if the marble lands in an odd space or on either of these two additional green spaces. This decreases the likelihood of a correct bet, increasing the likelihood that the casino will win money from the gambler. However, if the wheel included 0 but not 00, there would only be one additional space decreasing the player's chances of winning. As an extreme example, consider if there were 18 of the 0 spaces, the same number as the even and odd spaces. A bet on the marble landing in an even slot would have a 33% chance of success, compared with a 50% chance when there are no 0 spaces. A gambler is therefore more likely to bet correctly in European roulette, causing the casino to make less money than it would in American roulette.

A casino's advantage is more transparent in some games than in others. In poker, for example, there is a "rake," which is the amount each participant pays the casino in order to play (*Rake*, n.d.). There are different types of rakes that can be taken during a game. Oftentimes this is a rule set beforehand. One common type of rake takes a percentage of how much the players bet during the game, with a cap to limit the amount that must be paid. For example, the house may take 5% of what the players bet during the game, but only up to a rake of \$100. Other rakes are taken as entrance fees, membership fees or time fees (for example, \$20 for every hour you play). Rakes are taken in poker games because there is no house edge. All of the money bet during the game stays between the players. It is through rakes and by keeping players at the table as long as possible that casinos make their money in poker games.

Casinos are for-profit businesses. To ensure a profit, they design games and establish rules that lead to a positive house edge rather than a player advantage. There are, however, two ways that a gambler can achieve a player advantage. One way is by "counting cards" in blackjack, and the second way is by playing certain video poker games.

Counting cards requires covert mental calculations, which will be discussed after reviewing the rules of blackjack. The general rule in blackjack is to keep asking for cards until the sum of the card values is close to, but not over, 21. Face cards are valued at 10 points, and players can consider aces to be worth 1 or 11 points, whichever works to the player's advantage (Jones, n.d.). Although there may be several players at the table, players do not play against each other, they only play against the dealer. Still, each player should make note of the cards dealt to the other players. To start the game, each player will be dealt two cards face up. Only one of the dealer's

two cards will face up. Each player will ask for additional cards until they think they have enough to win, or they can “surrender” and receive half of their bet back. Unlike the players, the dealer has no choice about whether to take additional cards. The dealer must continue taking cards until the total score is 17 or more. If the total value of the cards is 16, there is a high risk of going over 21 with the next card; nevertheless, the dealer must take another card.

Given the rules of blackjack, knowledge about which cards have been dealt and which are still available would certainly give a player an advantage. Here is where counting cards comes into play. Hi-Lo is a card-counting strategy where “-1” is assigned to cards numbered two through six, “0” is assigned to cards numbered seven through nine, and “+1” is assigned to tens, face cards, and aces. As the cards are dealt to everyone at the table, a card-counting player can use this system to mentally calculate the sum for the table. For example, suppose one player had a 4 and a 10, a second player had a 7 and a jack, and the dealer’s face-up card was an ace. The count for the first player’s cards would be $-1+1=0$; the count for the second player’s cards would be $0+1=1$; and the count for the dealer’s card would be $+1$. The sum for the table would be $0+1+1=2$. For the next round, the count would start at 2. The count at the end of each round is used to start the counting for the next round.

A card-counting player will adjust the amount of the bet depending on how high or low the sum gets. If the sum is a high negative number, it means there have been many more low cards dealt than high cards, and there is a greater chance that the next card will be a high card. It follows then that if the dealer is at 16 and must take another card, there is a good chance that the dealer will go over 21 and lose. In this situation, it pays to bet high.

Conversely, a card-counting player will want to bet low if there is a higher positive sum. A high positive sum means that more high cards have been dealt already, so there is a greater chance that a lower card will be next. In this case, when the dealer is at 16 and must take another card, there is a good chance the dealer will draw a lower card like a 5, possibly hitting 21 and winning. When the sum is near 0, a card-counting player cannot predict the size of the next card with any confidence. The best route to take in this situation is to place a medium-sized bet.

The Hi-Lo strategy is the most popular card-counting strategy, but there are several other variations (“Is card counting illegal?” 2022). Some strategies set cards to different numbers, and some use values other than -1, 0, and +1. One of the most complex strategies is the “Halves” strategy. In this version of card-counting, “-1” is assigned to 10, face cards, and aces; “0” is assigned to 8; “+1” is assigned to 3, 4, and 6; -0.5 is assigned to 9; +0.5 is assigned to 2 and 7; and +1.5 is assigned to 5. Halves is a system geared more toward expert card counters but is one of the most accurate strategies and gives the player a larger advantage over the casino.

Card counting requires subtlety. If a casino employee suspects a player is using a card-counting strategy, the player may be asked to leave and may even be banned from playing at the casino. Successful use of card-counting strategies requires a great deal of practice. One must be able to focus on the calculations while maintaining a relaxed expression (“Is card counting illegal?” 2022).

An alternative route one can take to achieve a positive player advantage is to play certain kinds of video poker. Gamblers on these machines play variations of poker against the computer rather than other people, eliminating the variable of another player’s skill level. Three specific games that have a player advantage, when a player bets 5 credits, are Double Bonus Poker (0.172% player advantage), Double Double Bonus Poker (0.067%), and Deuces Wild (0.762%) (“Video

poker odds and strategy,” 2020). With these games, gamblers who play repeatedly can expect to win back a small percentage over what they bet, something that does not happen in other casino games. It may seem surprising that casinos would allow these games, given that a casino’s goal is to make money and these games have a player advantage, but the games have a betting cap. Though one could win a lot of money if large bets were permitted, small player advantages do not add up to much if only small bets are allowed.

For example, suppose a gambler plays Double Bonus Poker at about a 0.17% player advantage. One hour’s worth of gameplay, where the maximum amount is bet every time, would result in an estimated \$750 bet total. The multiplication of \$750 and 0.17% shows that one can expect to win about \$1.275 overall (Najera, 2022), an amount that means little to the gambler or the casino. Casinos offer some games with a small player advantage in the hope that gamblers will move on to games with a house edge.

It is now apparent that there are games that are statistically safer to play than others, and there are subcategories of games that can mitigate or exacerbate the advantage the casino has over the gambler. One can choose to play the safer games, yet most people are either unaware of the statistics involved or are aware of the risk but enjoy the thrill of a high-stakes game. Consequently, statistically dangerous games are among the most popular.

Players in the United States and Canada demonstrate a preference for many of the same games, including slot machines (between 4% and 15% house edge depending on the rules), American Roulette (2.7% to 7.89% house edge), blackjack (0% to 4% house edge), and Baccarat (1.06% to 1.24% house edge) (Barge, 2022) (“Ranking The 5 Most Popular Casino Games in Canada,” 2022). There are some differences, however, as players in the United States prefer craps (0% to 16.67% house edge) while Canadian players prefer video poker (-0.71% to 5% house edge), which is a safer game with a smaller house edge.

The tactics used by casinos can also be found outside their buildings in sports betting. Like casino games, sports betting contains an edge. The edge in sports betting benefits the bookmakers, who calculate the odds and handle the monetary transactions. While casinos conceal the house edge within the rules of the game, the edge in sports betting is much clearer.

There are three different types of odds that are used in sports betting. Each type will be demonstrated using the example of betting on which of two teams will win a game, with ties not permitted. The first type is fractional odds, also known as British or traditional odds (Kithinji, n.d.). In this type, the odds are shown as either a fraction or a pair of numbers having the word “to” between them, such as 3/5 or “3 to 5” odds. It means that on every successful \$5 bet, a gambler wins three additional dollars. The equation for this can be shown as:

$$\text{Winnings} = (\text{Numerator})(\text{Bet Amount}) \div (\text{Denominator})$$

If someone successfully bets \$10 on the 3/5 odds, the amount won would be $(3)(10) \div 5 = \$6$. Add on the \$10 that was bet in the first place, and the person will leave with \$16 in their pocket.

Fractional odds can also be used to show the likelihood of a team winning. This can be found using the equation:

$$\text{Fractional Odds Implied Probability} = (1 \div (\text{Fractional Odds} + 1)) \times 100$$

Using the example above of 3/5 odds, the team that is favored to win has a $(1 \div (3/5 + 1)) \times 100 = 62.5\%$ chance of winning. The team that is not favored to win has a $100\% - 62.5\% = 37.5\%$ chance of winning. The process can be reversed to calculate the fractional odds from the estimated probability of winning. This can be found using the equation:

$$\text{Fractional Odds} = (100 \div \text{Probability}) - 1.$$

If a team has a 62.5% chance of winning, the fractional odds would be calculated to be $(100 \div 62.5) - 1 = 3/5$.

A second type of odds is decimal odds, which is also known as European odds due to its popularity in Europe, Australia, New Zealand, and Canada (Sohail, 2023). This type of odds uses the equation: $\text{Total Return} = (\text{Stake})(\text{Decimal Odds})$, where total return is the amount of the bet plus the profit. The decimal odds show how much a person would win with a successful \$1 bet. If the odds of a team winning were 1.3, then a winning bet of \$1 would mean a total return of \$1.30. If the odds of a team winning were 3.50, then a winning bet of \$1 would mean a total return of \$3.50. Given the decimal odds, one can calculate the probability of a team winning using the equation:

$$\text{Decimal Odds Implied Probability} = (1 \div \text{Decimal Odds}) \times 100$$

Decimal odds of 3.00 would show that a team has a $(1 \div 3.00) \times 100 = 33.33\%$ chance of winning. Similarly, given the probability of a team winning, one can calculate the decimal odds using the equation:

$$\text{Decimal Odds} = 100 \div \text{Probability}$$

A team that has a 70% chance of winning would have decimal odds of $100 \div 70 = 1.43$ ("Bet Calculator and Odds Converter," n.d.).

Finally, the third type of odds is American odds. The key to American odds is the use of positive and negative signs to show who is predicted to win (-) and who is predicted to lose (+) ("How American Odds Work in Sports Betting," n.d.). Furthermore, the sign shows the way one should interpret the number following it. If it is a negative sign, then the number indicates how much one must bet to win \$100. If, on the other hand, it is a positive sign, then the number indicates how much one would win if they bet \$100.

For example, suppose Team A has odds of -150 and Team B, the opposing team, has odds of +140. Then someone betting \$150 on Team A will win \$100 if Team A wins, and someone betting \$100 on Team B will win \$140 if Team B wins. Finding the probability of winning from negative odds is:

$$\text{American Odds Implied Probability} = (\text{Negative Odds} \div (\text{Negative Odds} + 100)) \times 100$$

So, if the odds are -150, then the probability is $(150 \div (150 + 100)) \times 100 = 60\%$.

Finding the probability of winning from positive odds is:

$$\text{American Odds Implied Probability} = (100 \div (\text{Positive Odds} + 100)) \times 100$$

So, if the odds are +140, then the probability is $(100 \div (140 + 100)) \times 100 = 41.67\%$.

Finally, the American odds can be calculated from the probability of winning. The equations for the negative odds and positive odds are:

American Negative Odds = $(100 \times \text{Probability}) \div (100 - \text{Probability})$

American Positive Odds = $(10000 \div \text{Probability}) - 100$

Odds are put in these forms so that bookmakers can build in a profit for themselves, no matter who wins. In the above example, where the American odds are -150 and +140, the two probabilities add to 101.67%, rather than the expected 100%. Probabilities cannot go higher than 1.00 or 100%. This extra 1.67%, which is analogous to a house edge, is referred to as the “overround” (Sohail, 2023). Players are gambling on something that has a 100% chance of happening (that one of the teams will win), yet they are paying for their bets as if a 101.67% chance were possible. This can be thought of as a hidden “tax,” an additional cost being added to the value of the items being purchased. The added cost is subtly hidden in the numbers given to the gamblers, but that cost reliably gives the bookmakers their profit.

As another example, consider if both teams had 1.9 decimal odds, which is roughly a 52.6% chance for each team, so the probabilities would total 105.2%. If two people each bet one dollar on opposing teams, then two dollars are taken in. Once one of the teams wins, the winning gambler is given \$1.90 and the other gambler loses their \$1 bet. Thus, \$2 were bet, \$1.90 was given out as winnings, and \$0.10 was kept by the bookmakers.

Similar to the way casinos incorporate a house edge without much notice, the overround is easily concealed by putting it into odds form rather than reporting it as percentage. Saying that Team A has a 70% chance of winning and Team B has a 32% chance of winning shows clearly that something is not what it should be. However, when it is stated that the odds for Team A are -234 and the odds for Team B are +213, individuals who are unacquainted with the true meaning behind the numbers may not think twice about it.

CONCLUSIONS

A final note on this paper is that it takes a pessimistic view on gambling. It should be pointed out, though, that we have examined the topic through the lens of the maximum strategy, a concept in game theory where one tries to avoid the largest possible loss. The idea is to look at all the possible outcomes and choose whichever one appears to have the best end result, even if (and quite often when) that result is a loss (Barkley, 2021). With casinos, we would look at every game and recommend playing those with the smallest house edge, but that does not mean that one will assuredly lose money if they play the riskier ones. Chance can come in a person’s favor, and a game that may have a higher house edge could result in a large payout. Conversely, a game with a low house edge could result in a long losing streak that would lead one to lose more money there than if they played a riskier one. Mathematics can give one an expected outcome, but actual gameplay can vary greatly and can result in wins or losses that are drastically different from what was predicted. Another point that should be made is that financial gain is not the only reason people gamble. Gambling can be a social activity, a cognitive challenge, and an entertaining experience. With reasonable limits placed on expenditures of time and money, many individuals can safely enjoy placing a bet on a sports team or taking a trip to a casino.

In conclusion, there are many ways that gambling institutions can incorporate rules to ensure a profit. House edge is a hidden force within virtually every game in a casino, and statistics show that the average gambler will leave with an enjoyable experience and a lighter wallet. Sports gambling organizations use overround techniques to hide an impossible percentage chance that gamblers bet on, allowing the bookmakers to profit off the extra. Though a mathematical approach can inspire many intelligent ways to approach gambling, in the end statistics can only be used to estimate what will happen. It cannot predetermine whether a casino gambler or a sports bettor will ultimately win or lose money, but statistics can predict that the latter is more likely.

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